Charged Pion Polarizabilities to two Loops [1]

Urs Bürgi

Institut für Theoretische Physik
Universität Bern
Sidlerstrasse 5, CH-3012 Bern, Switzerland

(e–mail: buergi@butp.unibe.ch)

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Abstract

We evaluate the electric and magnetic polarizabilities of charged pions in the framework of chiral perturbation theory at next–to–leading order. This requires a two–loop evaluation of the Compton amplitude near threshold. We estimate the two new low–energy constants which enter the chiral expansion at this order with resonance saturation. The numerical results are compared with presently available experimental information.

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1. *Pion polarizabilities.* The first two terms in the threshold expansion of the Compton amplitude $\gamma\pi^\pm \to \gamma\pi^\pm$ involve the electric charge and the electric ($\bar{\alpha}_\pi$) and magnetic ($\bar{\beta}_\pi$) pion polarizabilities. To predict the values of these, we invoke chiral perturbation theory (CHPT) [1–5]. The resulting quark mass expansion of $\bar{\alpha}_\pi$ and $\bar{\beta}_\pi$ is very similar to the one of the threshold parameters in $\pi\pi$ scattering. For illustration, we consider, in addition to the polarizabilities, the chiral expansion of the $I = 0$, S–wave scattering length a_0. We have

$$
\begin{array}{cccc}
& \text{tree} & \text{1–loop} & \text{2–loops} \quad \text{3–loops}
\end{array}
$$

$$
\begin{aligned}
a_0 &= \frac{7\pi x}{2} \left\{ 1 \;+\; A\,x \;+\; B\,x^2 \;+\; O(M_\pi^6) \right\} , \\
\bar{\alpha}_\pi \pm \bar{\beta}_\pi &= \frac{\alpha}{M_\pi^3} \left\{ 0 \;+\; A_\pm\,x \;+\; B_\pm x^2 \;+\; O(M_\pi^4) \right\} , \\
x &= \frac{M_\pi^2}{16\pi^2 F_\pi^2} ,
\end{aligned}
\tag{1}
$$

where $F_\pi = 92.4$ MeV denotes the pion decay constant, and $\alpha = e^2/4\pi \simeq 1/137$. The first line indicates the number of loops that are needed to calculate the coefficients $A, A_\pm, \dots$. Whereas the scattering length starts out with a tree graph contribution [6], the leading order term in the polarizabilities is generated by one–loop diagrams [7–9]. The correponding numerical values are [7–9] [2]

$$
\bar{\alpha}_\pi \pm \bar{\beta}_\pi =
\begin{cases}
0 \\
5.4 \pm 0.8 \ .
\end{cases}
\tag{2}
$$

To estimate the reliability of this prediction, the corrections generated by two–loop diagrams must be evaluated [10, 11]. This has already been achieved in the case of the polarizabilities of the neutral pion [11]. In this letter we report the results for charged pions.

2. *Chiral expansion.* We first consider the threshold expansion of the Compton amplitude in powers of the photon momentum. In the rest frame of the incoming pion, we have

$$
T = -2 \left[\epsilon_1 \cdot \epsilon_2{}^*(e^2 - 4\pi M_\pi \bar{\alpha}_\pi |\mathbf{q_1}||\mathbf{q_2}|) - 4\pi M_\pi \bar{\beta}_\pi (\mathbf{q}_1 \times \epsilon_1) \cdot (\mathbf{q}_2 \times \epsilon_2{}^*) + \dots \right] ,
\tag{3}
$$

where $\mathbf{q}_i$ (ϵ_i) are the momenta (polarization vectors) of the photons. The ellipsis denotes higher order terms in the photon momenta. To arrive at the expansion Eq. (1), we consider the Compton amplitude in the framework of CHPT,

$$
T^{\mathrm{CHPT}} = T_2 + T_4 + T_6 + \cdots ,
\tag{4}
$$

where T_n is of the order $p^{(n-2)}$. This expansion is most conveniently performed in the framework of an effective lagrangian [1–5,10]. In the following we ignore isospin breaking effects by putting $m_u = m_d = \hat{m}$. The effective lagrangian is expressed in terms of the pion field U, the quark mass matrix χ and the external vector field v_μ,

$$
\mathcal{L}_{\mathrm{eff}} = \mathcal{L}_2(U, \chi, v_\mu) + \hbar\, \mathcal{L}_4(U, \chi, v_\mu) + \hbar^2\, \mathcal{L}_6(U, \chi, v_\mu) + \dots ,
\tag{5}
$$

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[2] We express the polarizabilities in units of $10^{-4}\mathrm{fm}^3$ throughout.

where $\mathcal{L}_n$ denotes a term of order p^n. The lagrangians $\mathcal{L}_2$ and $\mathcal{L}_4$ are given in Ref. [2], whereas the general structure of $\mathcal{L}_6$ has recently been determined in Ref. [12]. Given $\mathcal{L}_{\text{eff}}$, it is straightforward to expand the S–matrix elements in powers of $\hbar$. This procedure automatically generates the series (4). The expansion of the polarizabilities is obtained by comparing T^{CHPT} with the momentum expansion Eq. (3). The first term T_2 is due to tree graphs and describes Compton scattering from a point–like scalar particle. It does not contribute to the polarizabilities, because it reduces to the first term in the expansion Eq. (3). T_4 is responsible for the leading order term $A_\pm$ in (1). It contains two types of contributions: one–loop graphs generated by $\mathcal{L}_2$ and tree graphs involving one vertex from $\mathcal{L}_4$. The relevant momentum integrals have been performed in Ref. [13] – the corresponding numerical result for the polarizabilities is displayed in Eq. (2). Finally, the amplitude T_6 determines the next–to–leading corrections $B_\pm$. It contains two–loop contributions from the effective lagrangian $\mathcal{L}_2$ together with one–loop contributions from $\mathcal{L}_2 + \mathcal{L}_4$ and tree–level contributions from $\mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6$. We have found far more than 100 diagrams that contribute at this order. They are displayed in Refs. [14, 15].

3. *Next–to–leading order terms.* We omit the details of the calculation that is described in Refs. [14, 15]. Here we simply quote the results:

$$
\begin{aligned}
A_+ &= 0 , \quad A_- = \frac{2}{3}(\bar{l}_6 - \bar{l}_5) , \\
B_+ &= 8h^r_-(\mu) - \frac{4}{9}\left\{ l(l + \frac{1}{2}\bar{l}_1 + \frac{3}{2}\bar{l}_2) - \frac{53}{24}l + \frac{1}{2}\bar{l}_1 + \frac{3}{2}\bar{l}_2 + \frac{91}{72} + \Delta_+ \right\} , \\
B_- &= h^r_+(\mu) - \frac{4}{3}\left\{ l(\bar{l}_1 - \bar{l}_2 + \bar{l}_6 - \bar{l}_5 - \frac{65}{12}) \right. \\
&\qquad\qquad\qquad \left. -\frac{1}{3}\bar{l}_1 - \frac{1}{3}\bar{l}_2 + \frac{1}{4}\bar{l}_3 - (\bar{l}_6 - \bar{l}_5)\bar{l}_4 + \frac{187}{108} + \Delta_- \right\} , \\
l &= \ln \frac{M_\pi^2}{\mu^2} .
\end{aligned}
\tag{6}
$$

The quantities $\bar{l}_i$ denote low–energy constants from $\mathcal{L}_4$, and $h^r_\pm(\mu)$ are linear combinations of renormalized, scale–dependent low–energy couplings from $\mathcal{L}_6$. [The scale μ is introduced by dimensional regularization – the result for the polarizabilities is, of course, scale–independent.] Finally, the quantities $\Delta_\pm$ (generated partly by the two–loop integrals displayed in Fig. 1) are pure numbers, independent of low–energy constants and quark masses.

To obtain numerical results, we need the values of $\bar{l}_1, \ldots, \bar{l}_6 - \bar{l}_5, h^r_+$ and h^r_-. The constants $\bar{l}_i$ have been determined in Refs. [2, 16]. To obtain an estimate of the couplings $h^r_\pm$, we use resonance saturation [2, 11, 17, 18] with vector – and axialvector mesons ($J^{PC} = 1^{--}, 1^{+-}, 1^{++}$) [19–23]. We find (the uncertainties in the couplings quoted here are more generous than the ones given in Ref. [15])

$$
\begin{aligned}
h^r_+(M_\rho) &= 0.3 \pm 2.0 , \\
h^r_-(M_\rho) &= 0.45 \pm 0.15 .
\end{aligned}
\tag{7}
$$

4. *Numerics.* The numerical values of the polarizabilities are shown in Table 1 [3]. The one–

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[3] We use $F_\pi = 92.4$ MeV, $M_\pi = 139.6$ MeV and $\bar{l}_1 = -1.7 \pm 1.0$, $\bar{l}_2 = 6.1 \pm 0.5$, $\bar{l}_3 = 2.9 \pm 2.4$, $\bar{l}_4 = 4.3 \pm 0.9$, $\bar{l}_6 - \bar{l}_5 = 2.7 \pm 0.4$.

Table 1: Charged pion polarizability up to two–loops in units of 10^{-4}fm^3. The values in the brackets are included in the two–loop result (column 6).

	$\mathcal{O}(E^{-1})$	$\mathcal{O}(E)$			
	1–loop	$h^r_\pm$	$\Delta_\pm$	chiral logs	2–loops
$\bar\alpha_\pi + \bar\beta_\pi$	0.00	[0.15]	[0.16]	[0.04]	0.31
$\bar\alpha_\pi - \bar\beta_\pi$	5.36	[0.01]	[0.50]	[-1.45]	-0.94
$\bar\alpha_{\pi\pm}$	2.68	[0.08]	[0.33]	[-0.70]	-0.31
$\bar\beta_{\pi\pm}$	-2.68	[0.07]	[-0.17]	[0.75]	0.63

loop results are displayed in the second column, and the results of the two–loop calculation are presented in the third to sixth columns.

Comments:

– The total two–loop contributions displayed in the sixth column amount to a correction of 12%(24%) to the leading order value of $\bar\alpha_\pi(\bar\beta_\pi)$. We therefore expect higher order corrections to be substantially smaller.

– The contribution from the low–energy constants $h^r_\pm$ is substantial in the case of the sum $(\bar\alpha_\pi + \bar\beta_\pi)$, and smaller otherwise. The contribution from ρ–exchange alone, $(\bar\alpha_\pi + \bar\beta_\pi)_\rho = 0.07$, is more than three times smaller than the remaining two–loop contributions to this quantity.

– The fifth column contains the contributions from the chiral logarithms, i.e., the sum of the $\ln^2 M_\pi/\mu$ and $\ln M_\pi/\mu$ terms at the scale $\mu = M_\rho$. As has been shown in Ref. [1], the coefficients of the chiral *double logarithms* in any Green function can be determined using renormalization group arguments. We refer the reader to Ref. [11] for an illustration of the method in the neutral pion case, and to Ref. [24] for an application in elastic $\pi\pi$ scattering. In the present case, the method allows one to determine the coefficients $C_\pm$ in

$$
\begin{aligned}
(\bar\alpha_\pi \pm \bar\beta_\pi)_{2loops} &= C_\pm L_\chi + \dots , \\
L_\chi &= \frac{\alpha M_\pi}{(16\pi^2 F_\pi^2)^2} \ln(\frac{M_\pi^2}{\mu^2})\{\ln(\frac{M_\pi^2}{\mu^2}) + \frac{1}{2}\bar l_1 + \frac{3}{2}\bar l_2\} ,
\end{aligned}
\tag{8}
$$

with a one–loop calculation with $\mathcal{L}_2 + \mathcal{L}_4$. This serves as an additional check on the calculation. The result is $(C_+, C_-) = (-\frac{4}{9}, 0)$. The contribution of this term to the polarizabilities at $\mu = M_\rho$ is $(\bar\alpha_\pi, \bar\beta_\pi) = (0.16, 0.16)$.

– We found it interesting to investigate the effect of the genuine two–loop diagrams displayed in Fig. 1. These contributions are divergent at $d = 4$. Here we consider the finite pieces of these graphs, simply dropping the singular terms that occur in
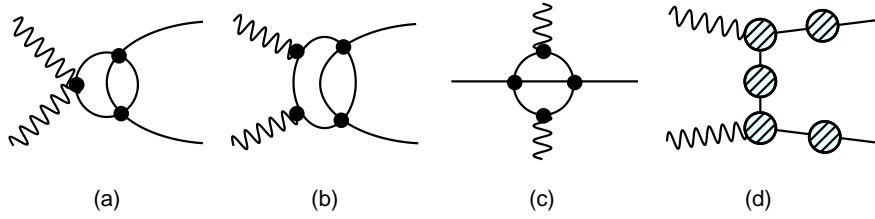
Figure 1: Two–loop graphs (vertex (a), box (b), acnode (c) and reducible diagrams (d)) generated by $\mathcal{L}_2$ which contain overlapping loop momenta and contribute to the Compton amplitude. A part of these contributions is included in the quantities $\Delta_\pm$. The hatched circles in diagram (d) denote self–energy and vertex corrections at one– and two–loop order.

Table 2: Logarithmic and total contributions of the two–loop diagrams depicted in Fig. 1 to the polarizabilities at the scale $\mu = M_\rho$ in units of $10^{-4}\mathrm{fm}^3$.

	box (b)			acnode (c)			reducible diagrams (d)		
	$\log$	$\log^2$	total	$\log$	$\log^2$	total	$\log$	$\log^2$	total
$\bar{\alpha}_\pi + \bar{\beta}_\pi$	-0.04	-0.45	-0.37	0	0	0.02	0.1	0	0.08
$\bar{\alpha}_\pi - \bar{\beta}_\pi$	-0.09	-1.34	-1.27	-0.48	-1.51	-2.13	-0.1	1.51	1.72

the renormalization scheme adopted in Ref. [15]. The results are given in Table 2, where we display the values of the logarithmic terms proportional to $\ln^2 M_\pi/M_\rho$ and to $\ln M_\pi/M_\rho$, together with the total contribution. It is seen that the chiral double logarithms are responsible for the large contribution to the polarizabilities.

– We have assumed that $h^r_\pm$ are saturated with resonance contributions at the scale of the rho mass. There is no particular reason to prefer this scale to say $\mu = 500$ MeV or $\mu = 1$ GeV. The corresponding results for the scales $\mu = 500, 700, 1000$ MeV are displayed in Table 3. It is seen that the sum $\bar{\alpha}_\pi + \bar{\beta}_\pi$ is nearly scale–independent, whereas the difference $\bar{\alpha}_\pi - \bar{\beta}_\pi$ changes by 20% by varying the scale from $\mu = 500$ MeV to μ=1 GeV.

End of comments.

Our final result for the polarizabilities of the charged pions at two–loop accuracy is

$$
\begin{aligned}
\bar{\alpha}_\pi + \bar{\beta}_\pi &= 0.3 \pm 0.1 & (0.0)\,, \\
\bar{\alpha}_\pi - \bar{\beta}_\pi &= 4.4 \pm 1.0 & (5.4 \pm 0.8)\,, \\
\bar{\alpha}_{\pi\pm} &= 2.4 \pm 0.5 & (2.7 \pm 0.4)\,, \\
\bar{\beta}_{\pi\pm} &= -2.1 \pm 0.5 & (-2.7 \pm 0.4)\,.
\end{aligned}
\tag{9}
$$

The numbers in brackets denote the leading order result. The estimates of the uncertainties stem from the uncertainties in the low–energy couplings $\bar{l}_1, \ldots, \bar{l}_6 - \bar{l}_5, h^r_\pm$ and do contain

Table 3: Scale–dependence of the polarizabilities in the resonance saturation scheme.

	$\mu = 500$ MeV	$\mu = 770$ MeV	$\mu = 1000$ MeV
$\bar{\alpha}_\pi + \bar{\beta}_\pi$	0.31	0.31	0.30
$\bar{\alpha}_\pi - \bar{\beta}_\pi$	4.94	4.42	4.11
$\bar{\alpha}_{\pi\pm}$	2.63	2.37	2.20
$\bar{\beta}_{\pi\pm}$	-2.32	-2.06	-1.90

neither effects from higher orders in the quark mass expansion nor any correlations. (The errors are more generous than quoted in Ref. [15].) In addition, the relation $(\bar{l}_6 - \bar{l}_5) = 6F_A/F_V$ [7] has been used [F_A and F_V denote axial and vector couplings in $\pi \to l\nu_l\gamma$.] This relation only holds at leading order in the chiral expansion – the implication of this fact for the uncertainties in (9) has not yet been worked out.

5. *Data versus CHPT.* The Compton amplitude near threshold has been extracted from photon–nucleus scattering $\gamma p \to \gamma \pi^+ n$ [25] and from radiative pion nucleus scattering (Primakoff effect) $\pi^- Z \to \pi^- \gamma Z$ [26]. Analyzing the data with the constraint $\bar{\alpha}_\pi + \bar{\beta}_\pi = 0$ gives

$$\bar{\alpha}_\pi - \bar{\beta}_\pi = \begin{cases} 40 \pm 24 & \text{(Lebedev [25])}, \\ 13.6 \pm 2.8 & \text{(Serpukhov [26])}. \end{cases} \tag{10}$$

Relaxing the constraint $\bar{\alpha}_\pi + \bar{\beta}_\pi = 0$, the Serpukhov data yield [27]

$$\begin{aligned} \bar{\alpha}_\pi + \bar{\beta}_\pi &= 1.4 \pm 3.1 \text{ (stat.)} \pm 2.5 \text{ (sys.)}, \\ \bar{\alpha}_\pi - \bar{\beta}_\pi &= 15.6 \pm 6.4 \text{ (stat.)} \pm 4.4 \text{ (sys.)}, \end{aligned} \tag{11}$$

where we have evaluated $\bar{\alpha}_\pi - \bar{\beta}_\pi$ from $\bar{\beta}_\pi$ and $\bar{\alpha}_\pi + \bar{\beta}_\pi$ as given in Ref. [27], adding the errors in quadrature.
The crossed amplitude $\gamma\gamma \to \pi^+\pi^-$ can be measured in pion–pair production in e^+e^-–collisions. At low energies, it is mainly sensitive to S–wave scattering. In Ref. [28] unitarized S–wave amplitudes have been constructed which contain $\bar{\alpha}_\pi - \bar{\beta}_\pi$ as an adjustable parameter. A fit to Mark II data at $E_{\pi\pi} < 800$ MeV [29] gives [28]

$$\bar{\alpha}_\pi - \bar{\beta}_\pi = 4.8 \pm 1.0. \tag{12}$$

(We have taken into account that the definition of the polarizability in Refs. [28, 30, 36] is 4π larger than the one used here). The result (12) contradicts the Serpukhov analysis Eq. (10). Taking into account also D–waves, Kaloshin et al. [30] find

$$\bar{\alpha}_\pi + \bar{\beta}_\pi = \begin{cases} 0.22 \pm 0.06 & (\text{ Mark II [29] }) \\ 0.30 \pm 0.04 & (\text{ CELLO [31] }). \end{cases} \tag{13}$$

A detailed analysis of the same data has also been performed in [32]. The authors conclude that the errors quoted in Eq. (13) are underestimated, see also Ref. [33]. By reanalyzing the present data on the angular distribution in $\gamma\gamma \to \pi^+\pi^-$, Kaloshin et al. [36] find

$$\bar{\alpha}_\pi + \bar{\beta}_\pi = \begin{cases} 0.22 \pm 0.07 \text{ (stat.)} \pm 0.04 \text{ (sys.)} & (\text{ Mark II [29] }) \\ 0.33 \pm 0.06 \text{ (stat.)} \pm 0.01 \text{ (sys.)} & (\text{ CELLO [31] }) \ . \end{cases} \tag{14}$$

Finally, we note that the optical theorem relates the sum of the polarizabilities to the total cross section $\sigma_{tot}^{\gamma\pi\pm}$. Writing an unsubtracted forward dispersion relation for one of the invariant amplitudes in $\gamma\pi^\pm \to \gamma\pi^\pm$ gives

$$\bar{\alpha}_\pi + \bar{\beta}_\pi = \frac{M_\pi}{\pi^2} \int_{4M_\pi^2}^{\infty} \frac{ds'}{(s' - M_\pi^2)^2} \sigma_{tot}^{\gamma\pi}(s') \ , \tag{15}$$

indicating that the sum of the polarizabilities has to be positive. Using a model for the total cross section, Petrun'kin [19] finds

$$\bar{\alpha}_\pi + \bar{\beta}_\pi = 0.39 \pm 0.04. \tag{16}$$

Note, however, that the lagrangian used [19] to estimate the cross section is not chiral invariant.

In order to clarify the experimental situation, new experiments to determine the pion polarizabilities have been planned at Fermilab (E781 SELEX), Frascati (DAΦNE), Grenoble (Graal facility) and at Mainz (MAMI). We refer the reader to the section on hadron polarizabilities in Ref. [35] for details.

We now comment on the comparison of the chiral predictions for $\bar{\alpha}_\pi \pm \bar{\beta}_\pi$ with the data.

– Our result $\bar{\alpha}_\pi - \bar{\beta}_\pi = 4.4 \pm 1.0$ includes the leading and next–to–leading order terms. It agrees within $1\frac{1}{2}$ standard deviation with the result 40 ± 24 found at Lebedev [25]. On the other hand, it is inconsistent with the value 13.6 ± 2.8 determined at Serpukhov [26].

– The analysis done by Kaloshin et al. [28, 30, 36] for $\bar{\alpha}_\pi \pm \bar{\beta}_\pi$ agrees within the error bars with the chiral predictions.

– The value $\bar{\alpha}_\pi + \bar{\beta}_\pi = 0.3 \pm 0.1$ includes the leading order term, generated by two–loop graphs. It is positive, in agreement with the sum rule (15), and in good agreement with the result (16).

6. *Summary.* In summary, we have presented the expression for the charged pion polarizabilities at next–to–leading order in the chiral expansion. In order to reduce the theoretical uncertainties, it would be necessary to determine the relevant low–energy constants with higher precision both, theoretically and experimentally. This requires e.g. the evaluation of the chiral corrections to radiative beta decay of the pion. Furthermore, it would be worthwhile to repeat the analysis of Petrun'kin [19] in the framework of chiral symmetry. On the experimental side, additional efforts are needed to clarify the situation.

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